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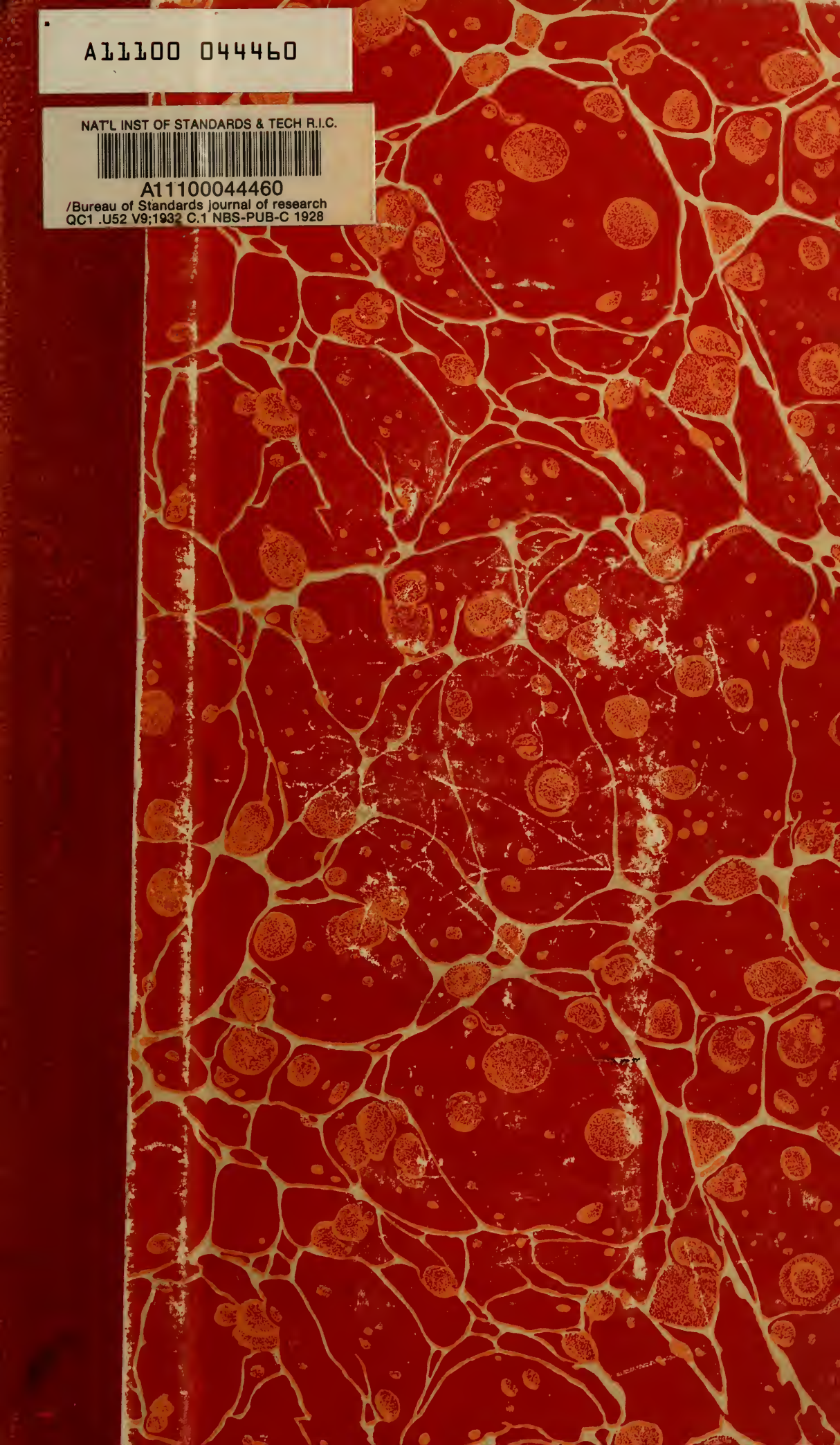
/Bureau of Standards journal of research
QC1 .U52 V9;1932 C.1 NBS-PUB-C 1928



CHARACTERISTIC EQUATIONS OF VACUUM AND GAS-FILLED TUNGSTEN-FILAMENT LAMPS

By L. E. Barbrow and J. Franklin Meyer

ABSTRACT

The manufacture of tungsten-filament incandescent lamps has changed very rapidly during recent years, and methods of lamp photometry have necessarily changed with the changes in the lamps. Photometric measurements have passed very largely from a mean horizontal candlepower basis to a lumen basis. The tables of characteristic relations of vacuum lamps, published in 1914 and based on measurements of horizontal candlepower, are no longer adequate for the newer types and larger sizes of lamps.

Logarithmic equations of the second degree are shown to apply to vacuum lamps and to gas-filled lamps in three steps. The equations describe the characteristics of miniature lamps as well as large lamps. Tables of characteristic relations, based on normal efficiencies of 10.0, 12.5, and 16.0 lumens per watt, computed by means of the characteristic equations, furnish means for ready calculation of light output, current, power input, and operating efficiency over a range of voltages from 55 per cent of normal voltage for vacuum lamps, and 80 per cent of normal voltage for gas-filled lamps, to 120 per cent of normal voltage for all types and sizes up to 150 watts, and to 132 per cent of normal voltage for large gas-filled lamps, sizes 200 watts and up.

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I. INTRODUCTION

In a previous Bureau of Standards publication[1] the derivation of characteristic equations of the form

$$y_n = A_n x^2 + B_n x + C \qquad (1)$$

from data obtained on vacuum tungsten lamps was fully described and applications of the equations to problems of the lamp standardizing and testing laboratory were illustrated. The equations were shown to express the relations of voltage to candlepower, to current, to watts input, and to the "efficiency," expressed in terms of watts per candle, over the range from 3.3 to 0.7 watts per candle. In these equations the variable x is the logarithm of the ratio of any voltage to that voltage at which a lamp operates when it emits light at an assumed

[1] Middlekauff and Skogland, Characteristic Equations of Tungsten Filament Lamps, B. S. Sci. Paper No. 238. Bull. B. S., vol. 11, p. 483, 1915.

"normal" efficiency. The variable y_n is the logarithm of the ratios of currents, candlepowers, or watts at these voltages, or the logarithm of "efficiency," expressed in watts per candle, at any voltage. A, B, and C are constants. Tables of the characteristic relations were included in the original paper. These have proved very useful in lamp testing and standardization, but need to be revised and extended in order to be applicable to lamps manufactured now. The necessary revisions and extensions are included in this paper.

II. THE LUMEN BASIS OF INCANDESCENT LAMP MEASUREMENT

1. CHANGES IN LAMP CONSTRUCTION

The earlier paper was published before the integrating sphere photometer had come into general use for measuring the total light output of lamps. Practically all lamps then had "squirrel-cage" filaments in clear bulbs and were rated by the manufacturers at so-called efficiencies expressed as watts per mean horizontal candle. Photometric measurements made on a horizontal bar photometer were therefore all that were required for the rating of lamps.

Within the last few years bulb sizes, shapes, and finishes have been changed, as have also the methods of mounting and supporting the filament. Inside-frosted, diffusing bulbs are used now on all sizes of lamps from 15 to 100 watts, and on many of the larger sizes of lamps. The ring-wound, coiled filament has replaced the "squirrel-cage" type of filament mounting. Besides, practically all lamps of 40 watts and above are now gas-filled instead of vacuum. These changes in the construction of lamps have caused changes in their characteristics which have been taken into account in the equations and tables of this paper.

2. PHOTOMETRY OF VACUUM AND GAS-FILLED LAMPS

It has long been known that horizontal candlepower measurements on gas-filled lamps present great difficulties and are generally unreliable.[2] Besides, for most illuminating engineering purposes a knowledge of the total light output of lamps rather than the candlepower in a particular direction is desired. Horizontal candlepower measurements on all types of lamps therefore have been given up almost entirely in commercial and testing-laboratory practice, and measurements of spherical candlepower or total flux have taken their place. In this paper the watts-per-candle basis of the characteristic equations given in Scientific Paper No. 238 has been replaced by a lumen-per-watt basis.

To make the equations of Scientific Paper No. 238 applicable to vacuum lamps when measured in lumens, conversion of mean horizontal candlepower values to lumen values is necessary. The spherical reduction factor, that is, the ratio of mean spherical candlepower to mean horizontal candlepower, for vacuum tungsten lamps, with "squirrel-cage" filament mountings, such as were used in 1914, is approximately 0.78. The normal basis for photometric measurements of vacuum tungsten lamps was formerly generally accepted as 1.20

[2] Middlekauff and Skogland, Photometry of Gas-Filled Lamps, Elec. World, Dec. 26, 1914, p. 1248; and B. S. Sci. Paper No. 264. Bull. B. S., vol. 12, p. 589, 1915–16. C. H. Sharp, Trans. Ill. Eng. Soc., vol. 9, p. 1021, 1914.

watts per mean horizontal candle. This normal value of 1.20 watts per candle becomes 8.17 lumens per watt $\left(\frac{0.78\times 4\pi}{1.20}=8.17\right)$ when the lamps are measured on a lumen basis. This is a much lower efficiency than that at which most vacuum tungsten lamps are now made, and consequently a value of 10.0 lumens per watt has been arbitrarily chosen, on which basis tabular values for vacuum tungsten lamps are calculated in this paper. The characteristics of clear vacuum tungsten lamps as derived in 1914 have been found to apply also to the inside-frosted vacuum lamps as made to-day, so that the equations and tables in this paper for vacuum lamps are fundamentally the same as those contained in Scientific Paper No. 238, but are modified from the candle to the lumen basis and the normal efficiency is increased from 8.17 to 10.0 lumens per watt. The equations for gas-filled lamps have different constants, but are of the same form as the equations for vacuum lamps.

The data from which were derived the equations for gas-filled lamps of sizes ranging from 200 watts up are the result of a very large number of measurements made over a period of 10 years at the Bureau of Standards and at the laboratories of manufacturers of incandescent lamps. The normal efficiency chosen for these equations is 16.0 lumens per watt. It has been found, however, that these equations do not fit the characteristics of the smaller sizes of gas-filled lamps. Two other sets of equations, one for gas-filled lamps of sizes 60 to 150 watts, and the other for gas-filled lamps of 40 and 50 watts have been derived. Both of these sets of equations are on a normal basis of 12.5 lumens per watt. It has been found, also, that the 200 to 500 watt equations and 60 to 150 watt equations describe the characteristics of 32 and 21 candlepower automobile headlamps, respectively, while the 40 to 50 watt equations are suitable for 3 candlepower, 6 to 8 volt lamps.

III. THE CHARACTERISTIC EQUATIONS OF INCANDESCENT LAMPS

The logarithmic form of characteristic equation as applied to vacuum lamps is completely discussed by Middlekauff and Skogland in their original paper,[3] page 494 and following. The form of equation for vacuum lamps:

$$y_n=A_nx^2+B_nx+C \qquad (n=1, 2, 3, 4) \qquad (1)$$

is found to apply to gas-filled lamps also. In this equation the independent variable x refers to voltage, and y_n and the corresponding constants A_n, B_n, and C to the efficiency in lumens per watt ($n=1$), light putput ($n=2$), power ($n=3$), and current ($n=4$). There are then four problems to which the general equation is applicable. The variable x is the logarithm of the voltage ratio in each case; that is

$$x=\log \text{ (voltage ratio)}$$

and

$$y_1=\log \text{ (lumens per watt)}$$
$$y_2=\log \text{ (light output ratio)}$$
$$y_3=\log \text{ (power ratio)}$$
$$y_4=\log \text{ (current ratio)}$$

[3] See footnote 1,

Constants A_n, B_n, and C have been computed on the selected normal bases of 10, 12.5, 12.5, and 16 lumens per watt, respectively, for vacuum lamps and for the three groups of different sizes of gas-filled lamps. These constants for the vacuum-lamp equations were derived from the constants contained in Bureau of Standards Scientific Paper No. 238, as previously indicated. For the various sizes of gas-filled lamps they were determined by setting up logarithmic curves of the relations, voltage ratio to luminous flux ratio, and voltage ratio to efficiency, using data obtained on several sizes of lamps measured over a wide range of voltages. The coefficients B_1 and B_2 were then determined as the slopes of these curves at the normal efficiency. The coefficients A_1 and A_2 were determined from the rates of change of these slopes. Coefficient C is the logarithm of the normal efficiency. Coefficients A_3, A_4, B_3 and B_4 are not independent of the above coefficients, A_3 and A_4 being equal to (A_2-A_1), B_3 being equal to (B_2-B_1), and B_4 being equal to (B_3-1).

TABLE 1.—*Constants in the characteristic equations*

General equation:

$$y_n=A_nx^2+B_nx+C \qquad (1)$$

in which x is log (voltage ratio), and y_n is the logarithm of the quantity given in the right-hand column of this table.

CONSTANTS

	Vacuum lamps	Gas-filled lamps			
	Size				
	15–60 w	40–50 w	60–150 w	200–500 w	Used to compute
	Normal lumens per watt				
	10.0	12.5	12.5	16.0	
A_1	−0.918	−1.482	−1.726	−1.690	Lumens per watt.
B_1	1.932	2.162	2.090	1.841	Do.
C[a]	1.00000	1.09691	1.09691	1.20412	Do.
A_2	−.946	−1.425	−1.669	−1.607	Light output (lumens) ratio.
B_2	3.513	3.685	3.613	3.384	Do.
A_3	−.028	.057	.057	.083	Power (watts) ratio.
B_3	1.5805	1.523	1.523	1.543	Do.
A_4	−.028	.057	.057	.083	Current (amperes) ratio.
B_4	.5805	.523	.523	.543	Do.

[a] C=0 for the other relations.

Example.—The lumens-per-watt equation for 500-watt, gas-filled lamps, with normal (100 per cent) value at 16.0 lumens per watt, is

$$y_1=-1.690x^2+1.841x+1.20412$$

where x is the logarithm of the ratio of any desired voltage to the voltage at normal efficiency (16.0 lumens per watt) and y_1 is the logarithm of the efficiency of the lamps at the desired voltage.

It is of interest to note that the coefficient of x (B_1, B_2, B_3, or B_4) in each case in Table 1 is very closely the exponent of the relationship existing between voltage and any one of the dependent variables listed in the last column of the table. For example, the efficiency of 500-watt lamps varies as the 1.84 power of the voltage; the luminous flux varies as the 3.38 power of the voltage; and the current varies as the 0.54 power of the voltage. These exponents are strictly applicable only for small changes of voltage near normal efficiency, but can be used through a wider range of voltages where only fairly close approximations are required.

There are 16 characteristic equations that can be written by using the constants of Table 1 to cover the four groups of lamp sizes, and the

four dependent characteristic variables of each group. It is not convenient, however, to use these equations directly in computations, because one usually does not know the voltage at which a particular lamp is to be operated to give the normal efficiency, and this must be determined before the equations can be used to give the value of any of the dependent variables at any desired voltage. To facilitate computations, Tables 2, 3, 4, and 5, which follow, have been set up by using the 16 characteristic equations, and their use is discussed below.

IV. TABLES OF CHARACTERISTIC RELATIONS OF VACUUM AND GAS-FILLED TUNGSTEN LAMPS

1. CHARACTERISTICS OF VACUUM TUNGSTEN LAMPS

TABLE 2.—*Characteristics of vacuum tungsten lamps, 15 to 60 watts (normal efficiency, 10 lumens per watt)*

$y_1 = -0.918x^2 + 1.932x + 1.000$ (used to obtain column 2)
$y_2 = -0.946x^2 + 3.513x$ (used to obtain column 3)
$y_4 = -0.028x^2 + 0.5805x$ (used to obtain column 4)
$y_3 = -0.028x^2 + 1.5805x$ (used to obtain column 5)

Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts	Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts
1	2	3	4	5	1	2	3	4	5
55	2.74	10.6	70.4	38.7	90	8.12	68.7	94.1	84.6
56	2.85	11.4	71.1	39.8	91	8.30	71.5	94.7	86.1
57	2.97	12.2	71.9	41.0	92	8.49	74.4	95.3	87.6
58	3.10	13.1	72.6	42.1	93	8.67	77.3	95.9	89.2
59	3.23	14.0	73.4	43.3	94	8.86	80.3	96.5	90.7
60	3.36	14.9	74.1	44.4	95	9.05	83.4	97.1	92.2
61	3.49	15.9	74.8	45.6	96	9.23	86.6	97.7	93.7
62	3.62	17.0	75.5	46.8	97	9.42	89.8	98.3	95.3
63	3.76	18.1	76.3	48.0	98	9.61	93.1	98.8	96.8
64	3.90	19.2	77.0	49.3	99	9.80	96.5	99.4	98.4
65	4.04	20.4	77.7	50.5	100	10.00	100.0	100.0	100.0
66	4.18	21.6	78.4	51.7	101	10.19	103.5	100.6	101.6
67	4.33	22.9	79.1	53.0	102	10.39	107.2	101.2	103.2
68	4.47	24.3	79.8	54.3	103	10.58	111.0	101.7	104.8
69	4.62	25.7	80.5	55.5	104	10.78	114.8	102.3	106.4
70	4.77	27.1	81.2	56.8	105	10.98	118.6	102.9	108.0
71	4.92	28.6	81.8	58.1	106	11.18	122.5	103.4	109.6
72	5.08	30.2	82.5	59.4	107	11.38	126.5	104.0	111.3
73	5.23	31.8	83.2	60.7	108	11.58	130.6	104.6	112.9
74	5.39	33.5	83.9	62.1	109	11.78	134.8	105.1	114.6
75	5.55	35.2	84.5	63.4	110	11.98	139.2	105.7	116.3
76	5.71	37.0	85.2	64.7	111	12.18	143.6	106.2	117.9
77	5.87	38.8	85.9	66.1	112	12.38	148.1	106.8	119.6
78	6.04	40.7	86.5	67.5	113	12.59	152.7	107.3	121.3
79	6.20	42.7	87.2	68.8	114	12.79	157.4	107.9	123.0
80	6.37	44.7	87.8	70.2	115	13.00	162.1	108.4	124.7
81	6.54	46.8	88.4	71.6	116	13.21	166.9	109.0	126.4
82	6.71	49.0	89.1	73.0	117	13.41	171.8	109.5	128.1
83	6.88	51.2	89.7	74.5	118	13.62	176.8	110.0	129.9
84	7.05	53.5	90.3	75.9	119	13.83	181.9	110.6	131.6
85	7.23	55.9	91.0	77.3	120	14.04	187.2	111.1	133.3
86	7.40	58.3	91.6	78.8					
87	7.58	60.8	92.2	80.2					
88	7.76	63.4	92.8	81.7					
89	7.94	66.0	93.4	83.1					

2. METHOD OF USING THE TABLES OF CHARACTERISTIC RELATIONS

The method of using these tables is as follows:

1. It is assumed that a lamp has been photometered and that at a given voltage V_0, the current A_0, and lumen output L_0 have been directly measured, and from these values, the watts, W_0, and lumens per watt, l_0 have been computed. It is required to find the current, A_1, lumens L_1, watts W_1, and lumens per watt, l_1, at the required voltage V_1.

2. The tables must be entered at the value of lumens per watt, l_0, or if l_0 is not a tabulated value, then at the value of l_0 interpolated into the table.

3. In the line of the table in which l_0 is found or interpolated (column 2 of each table) read or interpolate V_t, A_t, L_t, and W_t, which are the percentages of normal volts, amperes, lumens, and watts, respectively, for the lamp when operated at l_0 lumens per watt. Thus V_0 is V_t per cent of normal volts, A_0 is A_t per cent of normal amperes, L_0 is L_t per cent of normal lumens and W_0 is W_t per cent of normal watts.

4. The voltage V_1 at which it is required to determine the current, watts, lumens, and lumens per watt of the lamp is then $(V_1/V_0)\times V_t$ per cent of normal volts; that is, letting V'_t be the per cent of normal volts of the lamp at V_1 volts

$$V'_t = (V_1/V_0)\times V_t$$

In the line of the table containing V'_t or in an interpolated line, read or interpolate l'_t, L'_t, A'_t and W'_t, the lumens per watt, per cent of normal lumens, per cent of normal amperes, and per cent of normal watts, respectively, of the lamp when operated at V_1 volts.

Then

$$l_1 = l'_t \qquad = \text{lumens per watt at } V_1 \text{ volts}$$

$$L_1 = L_0(L'_t/L_t) = \text{lumens at } V_1 \text{ volts}$$

$$A_1 = A_0(A'_t/A_t) = \text{amperes at } V_1 \text{ volts}$$

$$W_1 = W_0(W'_t/W_t) = \text{watts at } V_1 \text{ volts}$$

also

$$W_1 = V_1 A_1$$

(a) NUMERICAL EXAMPLES ILLUSTRATING THE USE OF TABLES 2 TO 5

Example 1.—Given a vacuum tungsten lamp, which when burned at 115 volts, emits 243 lumens, and takes a current of 0.220 amperes. This lamp will hereafter in this example be referred to as the test lamp.

To find the lumen output, lumens per watt, current and power input of the test lamp at 124 volts.

Solution.—Find first by direct computation the watts and lumens per watt as follows:

Watts $= 115\times 0.220 = 25.3$.

Lumens per watt $= 243 \div 25.3 = 9.61$.

The known values of the lamp are then:

$V_0 = 115$ volts.

$A_0 = 0.220$ amperes.

$W_0 = 25.3$ watts.

$L_0 = 243$ lumens.

$l_0 = 9.61$ lumens per watt.

Enter table 2 (column 2) at 9.61 lumens per watt to determine the per cent of normal voltage at this efficiency. From column 1 of the line containing 9.61 lumens per watt we find that the lamp operates at 98 per cent of normal volts.

Since 115 volts is then 98 per cent of normal volts, 124 volts is (124/115)×98= 105.7 per cent of normal volts. The line of the table, interpolated (column 1) containing 105.7 per cent of normal volts contains also the lumens per watt and percentages of normal lumens, amperes, and watts of the lamp at 124 volts. By multiplying the known values of lumens, amperes, and watts at 115 volts by the ratios of the percentage values read from lines corresponding to 105.7 per cent and 98 per cent of Table 2, values at 124 volts are obtained,

The tabular statement below contains the complete calculations:

	Values read directly from table 2, 9.61 lumens per watt corresponding to 98 per cent volts	Values interpolated into table 2, at 105.7 per cent volts	Computation to 124 volts from columns 2 and 3	Computed values at 124 volts
1	2	3	4	5
Lumens per watt	9.61	11.12		11.12 lumens per watt.
	Per cent	*Per cent*		
Lumens	93.1	121.3	$\frac{243\times121.3}{93.1}$	317 lumens.
Amperes	98.8	103.25	$\frac{0.220\times103.25}{98.8}$	0.230 amperes.
Watts	96.8	109.1	$\frac{25.3\times109.1}{96.8}$	28.5 watts.

NOTE.—Column 5 gives the required values.

Example 2.—A 100-watt, gas-filled tungsten-filament lamp is measured at 125 volts and the following values are obtained:

V_0=125 volts.
A_0=0.912 amperes.
W_0=114 watts.
L_0=1,888 lumens.
l_0=16.56 lumens per watt.

Find amperes, watts, lumens, and lumens per watt at 100 volts. Table 4, page 728 is applicable in this example. Enter Table 4 to find in column 2, 16.56 lumens per watt. It is not found, but by interpolation between 16.49 and 16.76 we have for 16.56 lumens per watt, the following values:

Per cent normal volts $=115.26=V_t$.
Per cent normal lumens $=164.6=L_t$.
Per cent normal amperes $=107.7=A_t$.
Per cent normal watts $=124.2=W_t$.
Lumens per watt $=16.56=l_t$.

Then

$$V'_t=V_1/V_0\times V_t=100/125\times115.26=92.2$$

Enter table 4 at 92.2 per cent volts, interpolate and compute by using the ratios from lines 115.26 and 92.2 per cent normal volts in the table. We obtain then:

$$\text{Lumens}=1{,}888\times\frac{74.2}{164.6}=851.$$

$$\text{Amperes}=0.912\times\frac{95.9}{107.7}=0.812.$$

$$\text{Watts}=114\times\frac{88.4}{124.2}=81.1.$$

Lumens per watt=10.49.

3. CHARACTERISTICS OF GAS-FILLED TUNGSTEN LAMPS

TABLE 3.—*Characteristics of gas-filled lamps, 40 to 50 watts (also 3-candlepower, 6 to 8 volt lamps). Normal efficiency, 12.5 lumens per watt*

$y_1 = 1.482x^2 + 2.162x + 1.09691$ (used to obtain column 2)
$y_2 = 1.425x^2 + 3.685x$ (used to obtain column 3)
$y_4 = 0.057x^2 + 0.523x$ (used to obtain column 4)
$y_3 = 0.057x^2 + 1.523x$ (used to obtain column 5)

Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts	Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts
1	2	3	4	5	1	2	3	4	5
80	7.47	42.6	89.1	71.3	100	12.50	100.0	100.0	100.0
81	7.71	44.8	89.7	72.6	101	12.77	103.7	100.5	101.5
82	7.94	47.0	90.2	74.0	102	13.04	107.5	101.0	103.1
83	8.17	49.3	90.8	75.4	103	13.31	111.4	101.6	104.6
84	8.41	51.6	91.4	76.7	104	13.58	115.4	102.1	106.2
85	8.65	54.0	91.9	78.1	105	13.86	119.5	102.6	107.7
86	8.89	56.5	92.5	79.5	106	14.15	123.7	103.1	109.3
87	9.14	59.1	93.0	80.9	107	14.43	128.0	103.6	110.9
88	9.39	61.8	93.6	82.3	108	14.72	132.3	104.1	112.4
89	9.64	64.6	94.1	83.8	109	15.00	136.7	104.6	114.0
					110	15.28	141.3	105.1	115.6
90	9.89	67.4	94.7	85.2	111	15.56	145.9	105.6	117.3
91	10.14	70.3	95.2	86.6	112	15.85	150.7	106.1	118.9
92	10.39	73.3	95.8	88.1	113	16.13	155.5	106.6	120.5
93	10.65	76.3	96.3	89.6	114	16.41	160.4	107.1	122.1
94	10.91	79.4	96.8	91.0	115	16.70	165.4	107.6	123.8
					116	16.99	170.5	108.1	125.4
95	11.16	82.6	97.4	92.5					
96	11.43	86.0	97.9	94.0	117	17.27	175.7	108.6	127.1
97	11.70	89.4	98.4	95.5	118	17.56	180.9	109.1	128.8
98	11.96	92.8	99.0	97.0	119	17.85	186.3	109.6	130.4
99	12.22	96.3	99.5	98.5	120	18.15	191.8	110.0	132.1

TABLE 4.—*Characteristics of gas-filled lamps, 60 to 150 watts, inclusive (also 21-candlepower 6 to 8 volt lamps). Normal efficiency, 12.5 lumens per watt*

$y_1 = -1.726x^2 + 2.090x + 1.09691$ (used to obtain column 2)
$y_2 = -1.669x^2 + 3.613x$ (used to obtain column 3)
$y_4 = 0.057x^2 + 0.523x$ (used to obtain column 4)
$y_3 = 0.057x^2 + 1.523x$ (used to obtain column 5)

Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts	Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts
1	2	3	4	5	1	2	3	4	5
80	7.55	43.1	89.1	71.3	100	12.50	100.0	100.0	100.0
81	7.79	45.2	89.7	72.6	101	12.76	103.6	100.5	101.5
82	8.02	47.4	90.2	74.0	102	13.02	107.4	101.0	103.1
83	8.26	49.7	90.8	75.4	103	13.29	111.2	101.6	104.6
84	8.49	52.1	91.4	76.7	104	13.55	115.1	102.1	106.2
85	8.73	54.5	91.9	78.1	105	13.82	119.1	102.6	107.7
86	8.96	57.0	92.5	79.5	106	14.08	123.1	103.1	109.3
87	9.21	59.6	93.0	80.9	107	14.34	127.3	103.6	110.9
88	9.45	62.3	93.6	82.3	108	14.61	131.5	104.1	112.4
89	9.70	65.0	94.1	83.8					
					109	14.88	135.8	104.6	114.0
90	9.95	67.8	94.7	85.2	110	15.15	140.2	105.1	115.6
91	10.20	70.7	95.2	86.6	111	15.42	144.7	105.6	117.3
92	10.44	73.6	95.8	88.1	112	15.69	149.2	106.1	118.9
93	10.70	76.6	96.3	89.6					
94	10.95	79.7	96.8	91.0	113	15.96	153.8	106.6	120.5
					114	16.22	158.6	107.1	122.1
95	11.21	82.9	97.4	92.5	115	16.49	163.4	107.6	123.8
96	11.46	86.2	97.9	94.0	116	16.76	168.2	108.1	125.4
97	11.72	89.5	98.4	95.5					
98	11.98	92.9	99.0	97.0	117	17.04	173.2	108.6	127.1
99	12.24	96.4	99.5	98.5	118	17.31	178.3	109.1	128.8
					119	17.58	183.4	109.6	130.4
					120	17.84	188.6	110.1	132.1

TABLE 5.—*Characteristics of gas-filled lamps, 200 to 500 watts (also 32-candle-power, 6 to 8 volt, lamps). Normal efficiency, 16.0 lumens per watt*

$y_1 = -1.690x^2 + 1.841x + 1.20412$ (used to obtain column 2)
$y_2 = -1.607x^2 + 3.384x$ (used to obtain column 3)
$y_4 = 0.083x^2 + 0.543x$ (used to obtain column 4)
$y_3 = 0.083x^2 + 1.543x$ (used to obtain column 5)

Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts	Percentage of normal volts	Lumens per watt	Percentage of normal lumens	Percentage of normal amperes	Percentage of normal watts
1	2	3	4	5	1	2	3	4	5
80	10.23	45.4	88.7	71.0	105	17.49	117.8	102.7	107.8
81	10.51	47.5	89.3	72.4	106	17.78	121.5	103.2	109.4
82	10.79	49.7	89.9	73.7	107	18.07	125.3	103.8	111.0
83	11.07	52.0	90.5	75.1	108	18.36	129.2	104.3	112.6
84	11.35	54.3	91.0	76.5	109	18.66	133.2	104.8	114.2
85	11.64	56.6	91.6	77.9	110	18.95	137.2	105.4	115.9
86	11.92	59.1	92.2	79.3	111	19.23	141.3	105.9	117.5
87	12.21	61.6	92.8	80.7	112	19.52	145.4	106.4	119.2
88	12.50	64.2	93.3	82.1	113	19.81	149.6	106.9	120.8
89	12.78	66.8	93.9	83.6	114	20.10	153.8	107.4	122.5
90	13.07	69.5	94.5	85.0	115	20.4	158	108.0	124.2
91	13.36	72.2	95.0	86.5	116	20.7	162.5	108.5	125.8
92	13.65	75.1	95.6	88.0	117	21.0	167	109.0	127.5
93	13.94	78.0	96.2	89.4	118	21.2_5	172	109.5	129.2
94	14.24	80.9	96.7	90.9	119	21.5_5	176.5	110.0	130.9
95	14.53	83.9	97.3	92.4	120	21.8_5	181	110.5	132.6
96	14.82	87.0	97.8	93.9	121	22.1_5	186	111.0	134.4
97	15.12	90.2	98.4	95.4	122	22.4	190.5	111.5	136.1
98	15.41	93.4	98.9	96.9	123	22.7	195.5	112.0	137.8
99	15.70	96.6	99.5	98.5	124	23.0_5	200.5	112.5	139.6
100	16.00	100.0	100.0	100.0	125	23.2_5	205.5	113.0	141.3
101	16.30	103.4	100.5	101.5	126	23.5_5	210.5	113.5	143.1
102	16.60	106.9	101.1	103.1	127	23.8_5	215.5	114.0	144.8
103	16.90	110.5	101.6	104.7	128	24.1	221	114.5	146.6
104	17.20	114.1	102.2	106.2	129	24.4	226	115.0	148.4
					130	24.6_5	231.5	115.5	150.2
					131	24.9_5	237	116.0	152.0
					132	25.2	242.5	116.5	153.8

V. VERIFICATION OF THE CHARACTERISTICS

The upper range of efficiencies of 500-watt lamps was investigated several years ago in connection with tests made for the optical division of the Bureau of Standards. Starting with a basic value of 17.6 lumens per watt at 105 volts, the following comparison of observed and computed values is an example of the results obtained.

TABLE 6.—*Efficiency of 500-watt lamps*

Volts	Efficiency in lumens per watt	
	Observed	Computed
90	13.2	13.2
100	16.1	16.1
105	17.6	
110	19.1	19.1
120	21.9	22.0
125	23.6	23.4
135	26.3	26.2

Values were recorded for 5-volt steps upward to burn out, which occurred within the step from 195 to 200 volts. Departures from computed values were not very extreme even at the higher points; for example, the observed value at 175 volts was 2.5 per cent below the computed value, and at 185 volts the corresponding difference was 2 per cent. A prohibitive amount of work would be required to assign characteristic values to these extreme points, where the observations have to be taken rapidly against great odds of color difference. Consequently, no attempt has been made to include them in the derivation of equations, and the lumens per watt in Table 5 terminate at 25.2.

Extrapolations have been made to points considerably below the limits of the tables for gas-filled lamps in work done on 500-watt lamps. In this work the voltage and light output at color match with and in terms of vacuum tungsten lamps were determined. At this color the 500-watt lamps operated at about 6.7 lumens per watt. Determinations of light output were then made at voltages corresponding to 16.5 lumens per watt in terms of gas-filled standards. Computed from the voltage and light output for color match with vacuum tungsten standards to these upper voltages, the values of light output checked the values obtained by observation to within 0.3 per cent.

The entire range of all the tables has been explored by means of a physical photometer (phototube photometer). A gas-filled cæsium phototube, corrected for color by means of a suitable filter, and attached to a 60-inch integrating sphere, was used. The phototube circuit is similar to that devised by Sharp and Smith.[4] This phototube photometer has been used at the Bureau of Standards for over a year in the rating of lamps for life test. Every month the characteristics of the photometer are checked by reading on it three of each of seven sizes of standard lamps ranging in size from 15 to 100 watts. The color temperature range of the light from these lamps is approximately 2,400° to 2,800° K. Each time there has been a remarkably close check between the readings thus obtained and the values assigned to the lamps by visual photometry using an 88-inch integrating sphere and Lummer-Brodhun photometer head. There have been other checks on the phototube photometer from time to time and these have shown the photometer to be accurate over a large range of color temperature and flux (lumen output).

All regular sizes of lamps from 15 to 500 watts, and also miniature lamps of 3, 21, and 32 candlepower have been photometered at various voltages to cover the range of the tables of this paper. These lamps were selected at random from the lamps received for life test from five different lamp manufacturers. Typical sets of results are given in Table 7.

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[4] Sharp and Smith, Trans. Ill. Eng. Soc., vol. 23, p. 434, 1928.

TABLE 7.—*Observed and computed values of lumens and amperes on lamps of various sizes*

(*a*) AVERAGE OF TWO 15-WATT, VACUUM LAMPS (TABLE 2 USED TO OBTAIN COMPUTED VALUES)

Volts	Lumens		Amperes	
	Photo-tube	Com-puted	Potenti-ometer	Com-puted
80	34.7	34.2	0.106	0.107
90	54	54	.114	.114
100	79	80	.122	.122
115	132	---	.132	---
125	176	176	.139	.139
130	202	204	.142	.142

(*b*) AVERAGE OF TWO 40-WATT GAS-FILLED LAMPS (TABLE 3 USED TO OBTAIN COMPUTED VALUES)

100	252	254	0.332	0.330
110	366	366	.346	.346
120	508	510	.364	.362
125	592	---	.370	---
135	784	784	.384	.385
145	1,005	1,008	.400	.400

(*c*) AVERAGE OF TWO 3-CANDLE-POWER GAS-FILLED LAMPS (TABLE 3 USED TO OBTAIN COMPUTED VALUES)

6.72_5	34.2	---	0.500	---
7.50	51.2	51.4	.527	0.528
8.00	64.7	64.8	.544	.547
8.5	79.8	80.6	.560	.564

(*d*) AVERAGE OF TWO 100-WATT, GAS-FILLED LAMPS (TABLE 4 USED TO OBTAIN COMPUTED VALUES)

90	572	570	0.766	0.767
100	852	852	.810	.811
115	1,411	---	.872	---
125	1,888	1,888	.912	.911
130	2,158	2,150	.931	.931

(*e*) AVERAGE OF TWO 21-CANDLE-POWER GAS-FILLED LAMPS (TABLE 4 USED TO OBTAIN COMPUTED VALUES)

5.00	110	109	2.15	2.15
5.50	156	155	2.26	2.26
6.00	211	211	2.36	2.36
6.35	256	---	2.44	---

(*f*) AVERAGE OF TWO 200-WATT, GAS-FILLED LAMPS (TABLE 5 USED TO OBTAIN COMPUTED VALUES)

90	1,208	1,216	1.425	1.424
100	1,778	1,786	1.507	1.507
115	2,912	---	1.624	---
125	3,868	3,847	1.699	1.700
129	4,299	4,285	1.728	1.730

(*g*) AVERAGE OF TWO 32-CANDLE-POWER, GAS-FILLED LAMPS (TABLE 5 USED TO OBTAIN COMPUTED VALUES)

4.00	92	90	2.91	2.89
4.50	142	141	3.08	3.08
5.00	206	206	3.25	3.24
5.50	288	288	3.42	3.42
6.02_5	390	---	3.59	---

VI. CONCLUSION AND ACKNOWLEDGMENTS

The equations and tables embodied in this paper have been used for several years at the Bureau of Standards in connection with the routine testing of incandescent lamps. For the various sizes of lamps, slide rules have been devised and these are used to determine the voltages at which lamps are to be burned in order to operate at any desired efficiency, or the operating efficiency of the lamps when burned at any designated voltage. Although this is the general use of the equations and tables here, they have found other occasional uses, such as for determining the voltages at which to calibrate lamps when a designated lumen output or efficiency of the lamps is required. The results obtained by these computations have been checked many times on a visual photometer by groups of experienced observers.

The authors desire to acknowledge the work of the late J. F. Skogland in connection with this paper. He had much of the data here presented nearly ready for publication when he died. It was felt, however, that the equations should be further checked before publication, and this has been done. We wish to state also that the data from which the 200 to 500 watt lamp equations were derived were obtained by cooperative work between the Nela Park laboratories of the General Electric Co. and the Bureau of Standards.

Washington, August 16, 1932.